Robustification of Learning Observers to Uncertainty Identification via Time-varying Learning Intensity

Chengxi Zhang, Choon Ki Ahn, *Senior Member, IEEE*, Jin Wu, Wei He, *Senior Member, IEEE*, Yi Jiang, and Ming Liu, *Senior Member, IEEE*

***Abstract*—This brief studies the simultaneous estimation of states and uncertainties in general continuous-time systems. In particular, we present a novel time-varying learning intensity (TLI) learning observer (LO). It has the advantage of inheriting the valuable properties of conventional LOs with a simple structure, *i.e.*, the uncertainty estimation is achieved using simply one algebraic equation with low computational costs. The foremost difference in comparison with conventional LOs is the utilization of the TLI approach, which attenuates the overshooting response in the case of large estimation errors and obtains decent performance improvement. Simulations for constant and time-varying signals demonstrate a notable performance boost of TLI-LO.**

***Index Terms*—Learning observer, time-varying learning intensity, uncertainty estimation.**

I. Introduction

A. Background and Motivation

THE dynamical model of a general linear system with system uncertainty is described by [1]

$$\dot{x}(t) = Ax(t) + Bu(t) + Ef(t) \tag{1a}$$

$$y(t) = Cx(t) \tag{1b}$$

where $x(t) \in \mathbb{R}^{n\times 1}$ denotes the system state, and $y(t) \in \mathbb{R}^{p\times 1}$ denotes the *measurable* output; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $E \in \mathbb{R}^{n\times q}$, and $C \in \mathbb{R}^{p\times n}$ are constant matrices; $u(t) \in \mathbb{R}^{m\times 1}$ denotes the input signal; $f(x) \in \mathbb{R}^{q\times 1}$ represents the uncertainty signal induced by internal and external disturbances (also called faults). Using an observer is an efficacious technique to estimate the state and uncertainty in (1) [2]. The estimation results are utilized to participate in the control design, improving the controller's robustness to cope with the system's unpredictable uncertainty item and thus improve the control performance. Compared with the adaptive approach estimating the perturbation's upper bound to compensate in the controller, the observer approach reduces the controller's conservativeness. Among various observer designs, the LO is an attractive research topic, with the advantages of simple structure with low computational demands and without the need for derivatives of the estimated signal to be available everywhere [3]. In this brief, we are interested in designing an effective LO with TLI for a system given by (1).

C. Zhang is with Harbin Institute of Technology, Shenzhen, China (e-mail: dongfangxy@163.com).

C. K. Ahn is with the School of Electrical Engineering, Korea University, Seoul 136-701, South Korea (e-mail: hironaka@korea.ac.kr).

J. Wu and M. Liu are with the Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong, China (e-mail:eelium@ust.hk).

W. He is with Institute of Artificial Intelligence, University of Science and Technology Beijing, Beijing 100083, China.

Y. Jiang is with Department of Mechanical and Biomedical Engineering, College of Science and Engineering, City University of Hong Kong, Hong Kong, China.

(*Corresponding author: Chengxi Zhang; Choon Ki Ahn.*)

B. Related Work

The structural simplicity and low computational demand of the LO come from the fact that it only uses a single algebraic equation to estimate the uncertainty, without any other operations such as derivatives, integrals, or other more complex computations. For example, in [3, Eq. (2)], the uncertainty term is estimated by

$$\hat{f}(t) = K_1\hat{f}(t-\tau) + K_2(y(t) - \hat{y}(t)) \tag{2}$$

with estimations $\hat{f}(t)$ for $f(t)$ and $\hat{y}(t)$ for $y(t)$ in (1); both K_1 and K_2 are constant matrices. The first item in (2) on the right side of the equals sign can be called the *learning item*, representing the learning of previous information, which is the most apparent feature of the LO; $\tau > 0$ is called the *learning interval*; $K_1 = \text{diag}(k_{1,1}, ..., k_{1,q})$ is called the *learning intensity*, which represents the weight of the information $\hat{f}(t-\tau)$ updated to the latest estimate $\hat{f}(t)$; the second item to the right of the equals sign is called the *updating item*, which represents the update of the latest output error information into the estimation.

This simple and effective strategy is so effective that various investigations have been published, for example, [3]–[6] used this approach to explore different systems fruitfully. Nevertheless, there is a quite noticeable problem that has not yet been explored in-depth, which is that, the K_1 in existing investigations has been defined as *fixed* values. This has resulted in the learning intensity being the same for all observers regardless of the situation. This leads to the problem that fast signal reconstruction cannot be achieved when the learning intensity is low. In contrast, when the learning intensity approaches 100%, although the estimation can be accomplished swiftly, it incurs notable *chattering responses (or overshooting)* in the case of large output errors. There are some evident problems related to the results, such as [5, Figs. 1-3], [4, Figs. 5 and 6], [3, Figs. 1-4], [7, Figs. 11-14]. The choice is made to select a high learning intensity for fast estimation purposes in these current results.

Therefore, further attenuating the chattering response will significantly enhance the LO's performance, which becomes the exploration key-point in this work.

C. Contributions

The main contributions of the novel TLI-LO are threefold.

1) A novel TLI-LO is designed that employs a TLI approach to tune the observer's update weights when the output error varies. Compared with the fixed learning intensity approaches in [3]–[8], the chattering phenomenon is significantly attenuated.
2) Compared with existing studies such as [4, Assumption 4], [7, Assumption 1] and [6, Assumption 3], we provide a more general form of the demands on the signal to be estimated, *i.e.*, conventional assumptions can be directly derived from its bounded properties.
3) The proposed TLI-LO maintains the advantages of the learning-type observers with a simple structure, low computational demands, and fast estimation of uncertainty. Moreover, the globally asymptotic stability w.r.t. a set can be guaranteed.

II. Preliminaries

The system investigated is given by (1). Furthermore, we have added the following Assumptions.

Assumption 1. $\text{rank}(E) = q$ and (A, C) is detectable.

Assumption 2. $\|f(t)\|$ is bounded, *i.e.*, $\|f(t)\| \le \varepsilon$.

From $\hat{y}(t)$, $\hat{f}(t)$ in (2), define the estimation (identification) errors for $y(t)$ and $f(t)$,

$$e_y(t) := y(t) - \hat{y}(t) \tag{3a}$$
$$e_f(t) := f(t) - \hat{f}(t). \tag{3b}$$

Define an aided variable

$$\tilde{f}(t) := f(t) - K(t)f(t-\tau) \tag{4}$$

with $\tau > 0$ and $\|K(t)\|$ is bounded.

From Assumption 2, we have $f(t) = \beta(t)f(t-\tau)$ where obviously $\beta(t) \in [-1, 1]$. Therefore, we have $\|\tilde{f}(t)\| = \|f(t) - K(t)f(t-\tau)\| = \|f(t-\tau)(\beta(t)I - K(t))\| \le \varepsilon\|\beta(t)I - K(t)\| < \varepsilon(\|\beta I\| + \|K(t)\|) < \vartheta$. Then, the Lemma 1 can be a corollary, and is a general form of the conditions described in studies [4, Assumption 4], [6, Assumption 3].

Lemma 1. $\|\tilde{f}(t)\|$ is bounded if Assumption 2 holds, with $\|\tilde{f}(t)\| < \vartheta$.

Lemma 2 [6]. For two vectors $\mathfrak{a}$ and $\mathfrak{b}$ with proper dimensions, a positive parameter α exists such that

$$\mathfrak{a}^\top \mathfrak{b} + \mathfrak{b}^\top \mathfrak{a} \le \frac{1}{\alpha}\mathfrak{a}^\top \mathfrak{a} + \alpha \mathfrak{b}^\top \mathfrak{b}. \tag{5}$$

Remark 1. Note that Assumption 2 is a sufficient condition. A weak requirement is that $\tilde{f}(t)$ is bounded, which can be obtained if $f(t)$ satisfies the Lipschitz condition. However, the Lipschitz condition requires a function to be *smooth*, while the bounded condition does not require the derivative to exist everywhere. That is, Lipschitz condition is conservative. Since uncertainty signals are mostly induced by modeling uncertainties, assembly errors, and actuator failures in practice, they are bounded in engineering. Hence the requirement in Assumption 2 is reasonable.

Remark 2. Some adaptive observers require that the uncertainty signal and its time derivative are bounded [4, Remark 4]. This paper does not require that the derivatives exist everywhere *i.e.*, only $\|f(t)\|$ is bounded, which is a much less constrained necessity. Lemma 1 shows that we can obtain the assumptions needed in conventional LOs via Assumption 2.

Remark 3. The more general meaning of Lemma 1 is that it does not hold in relation to K_1 as stated in [4] and [6], *i.e.*, as long as $f(t)$ is bounded, Lemma 1 naturally holds.

III. Time-varying learning intensity LO design

A. Observer Design

The proposed TLI-LO is described by

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + E\hat{f}(t) + Le_y(t) \tag{6a}$$
$$\hat{y}(t) = C\hat{x}(t) \tag{6b}$$
$$\hat{f}(t) = K(t)\hat{f}(t-\tau) + K_2 e_y(t) \tag{6c}$$

with estimation $\hat{x}(t)$ for $x(t)$. Define

$$e_x(t) := x(t) - \hat{x}(t), \tag{7}$$

and (3b) becomes

$$e_y(t) = C(x(t) - \hat{x}(t)) = Ce_x(t), \tag{8}$$

$L \in \mathbb{R}^{n\times p}$ and $K_2 \in \mathbb{R}^{q\times p}$ are the gain and updating intensity matrices; $K(t)$ is the learning intensity matrix, which is a *time-varying* matrix defined by $K(t) = \text{diag}(k_1(t), ..., k_i(t), ..., k_q(t))$ where $i \in \{1, ..., q\}$ and each element in $K(t)$ is given by

$$k_i(t) = \exp\left[-\lambda_1(\|\bar{e}_{f,i}\| + \|\bar{e}_y\|)^{\lambda_2} - \gamma\right] \tag{9}$$

where $\lambda_1 > 0$, $\lambda_2 > 0$ and $\gamma > 0$ are constants; $\bar{e}_y := \frac{1}{\tau}\left[e\left(t\right) - e\left(t-\tau\right)\right]$; $\bar{e}_{f,i} = \frac{1}{\tau}[\hat{f}_i\left(t\right) - \hat{f}_i\left(t-\tau\right)]$.

Combining (3b) and (6c) yields

$$\begin{aligned} e_f(t) &= f(t) - \hat{f}(t) \\ &= f(t) - K(t)f(t-\tau) + K(t)f(t-\tau) \\ &\quad - K(t)\hat{f}(t-\tau) - K_2 e_y(t) \\ &= K(t)e_f(t-\tau) - K_2 e_y(t) + \tilde{f}(t) \\ &= K(t)e_f(t-\tau) - K_2 Ce_x(t) + \tilde{f}(t). \end{aligned} \tag{10}$$

From (1), (8), and (6c), it yields

$$\begin{aligned} \dot{e}_x(t) &= Ae_x(t) + Ee_f(t) + Le_y(t) \\ &= Ee_f(t) + (A - LC)e_x(t). \end{aligned} \tag{11}$$

Remark 4. From (9), we know that $k_i(t)$ is monotonically decreasing with respect to $e_y(t)$. In (9), γ ensures that $k_i(t) \ne 1$ at $\|\bar{e}_{f,i}\| + \|\bar{e}_y\| = 0$, and $k_i(t) \to 0$ only at $\|\bar{e}_{f,i}\| + \|\bar{e}_y\| \to \infty$, hence $k_i(t) \in (0, 1)$. Therefore $\|K(t)\|$ is obvious bounded. λ_1, λ_2, and γ together shape the VLI function, which is bell-shaped. λ_1 affects the decay rate away from 0, and the larger it is, the faster the function decays; λ_2 determines the degree of flatness of $k_i(t)$ near 0, and the larger it is, the flatter the function is in this neighborhood; γ limits the maximum $k_i(t)$ and prevents it from reaching 1.

Remark 5. The symbol $\bar{e}_y$ is used to describe the intensity of its variation, which approximates the derivative's definition. By this arrangement, we can see that the proposed TLI approach reduces the learning intensity when the estimation error and variability are large. Using this approach, we expect to obtain a smoother estimation result.

Remark 6. Eq. (9) presents our proposed TLI method for LO, which is the most significant distinction between the TLI-LO designed in this paper and those in [3]–[6]. We have advanced the learning intensity in a way that varies with respect to the estimation errors. Clearly (9) shows that the learning intensity will be strengthened when the estimation error tends to zero and be weakened when the estimation error is large. The TLI approach will significantly improve the fixed learning intensity issues in conventional approaches, *i.e.*, when the learning intensity is close to 1, but chattering occurs when the error is in a large situation. Using this approach, we expect LO to estimate state and uncertainty signals more smoothly when the error is large but still maintain a high estimation accuracy when the error becomes small.

B. Stability Analysis

Proposition 1. For the system described by (1) and satisfying Assumptions 1 and 2, the TLI-LO given by (6) can achieve the simultaneous estimation of state and uncertainty as long as the following conditions hold:

C1) $\Omega E = \mu (K_2 C)^\top$;
C2) $(A-LC)^\top\Omega + \Omega(A-LC) - (\mu-\phi_2)(K_2C)^\top K_2 C < 0$;
C3) $\Xi - (\mu-\phi_1)K^\top(t)K(t) > 0$;

where $\Omega \in \mathbb{R}^{n\times n}$ and $\Xi \in \mathbb{R}^{q\times q}$ are positive definite matrices; $\mu = (\xi+1)\lambda_{\max}(\Xi)$, and $\lambda_{\max}(\cdot)$ is the maximum eigenvalue of corresponding matrix; $\xi > 0, \phi_1 > 0, \phi_2 > 0$.

Proof: Select the following positive definite function for TLI-LO

$$V(t) = e_x^\top(t)\Omega e_x(t) + \int_{t-\tau}^{t} e_f^\top(\chi)\Xi e_f(\chi)d\chi \tag{12}$$

such that $V(t) \geq 0$ holds. Then its time derivative satisfies

$$\begin{aligned}\frac{dV(t)}{dt} =& e_f^\top(t)E^\top\Omega e_x(t) + e_x^\top(t)(A-LC)^\top\Omega e_x(t)\\ &+ e_x^\top(t)\Omega E e_f(t) + e_x^\top(t)\Omega(A-LC)e_x(t)\\ &+ e_f^\top(t)\Xi e_f(t) - e_f^\top(t-\tau)\Xi e_f(t-\tau)\\ =& e_x^\top(t)\left[(A-LC)^\top\Omega + \Omega(A-LC)\right]e_x(t)\\ &+ 2e_x^\top(t)\Omega E e_f(t) + e_f^\top(t)\Xi e_f(t)\\ &- e_f^\top(t-\tau)\Xi e_f(t-\tau)\\ =& e_x^\top(t)\Pi e_x(t) + 2e_x^\top(t)\Omega E e_f(t) + e_f^\top(t)\Xi e_f(t)\\ &- e_f^\top(t-\tau)\Xi e_f(t-\tau)\end{aligned} \tag{13}$$

with $\Pi = (A-LC)^\top\Omega + \Omega(A-LC)$. Since

$$\xi e_f^\top(t)\Xi e_f(t) \leq e_f^\top(t)[\xi\lambda_{\max}(\Xi)]e_f(t) \tag{14}$$

where $\xi > 0$ is a constant. Therefore, (13) satisfies

$$\begin{aligned}\frac{dV(t)}{dt} \leq& e_x^\top(t)\Pi e_x(t) + 2e_x^\top(t)\Omega E e_f(t)\\ &+ (\xi+1)e_f^\top(t)\lambda_{\max}(\Xi)e_f(t)\\ &- e_f^\top(t-\tau)\Xi e_f(t-\tau)\\ &- \xi e_f^\top(t)\Xi e_f(t).\end{aligned} \tag{15}$$

Consider (10), we have

$$\begin{aligned}\mu e_f^\top(t)e_f(t) =& \mu e_f^\top(t-\tau)K^\top(t)K(t)e_f(t-\tau)\\ &+ \mu e_x^\top(t)(K_2C)^\top K_2 C e_x(t)\\ &- 2\mu e_x^\top(t)(K_2C)^\top K(t)e_f(t-\tau)\\ &+ 2\mu\tilde{f}^\top(t)K(t)e_f(t-\tau)\\ &- 2\mu\tilde{f}^\top(t)K_2Ce_x(t).\end{aligned} \tag{16}$$

Furthermore,

$$\begin{aligned}2e_x^\top(t)\Omega E e_f(t) =& 2e_x^\top(t)\Omega E K(t)e_f(t-\tau)\\ &- 2e_x^\top(t)\Omega E K_2 C e_x(t)\\ &+ 2e_x^\top(t)\Omega E\tilde{f}(t).\end{aligned} \tag{17}$$

Considering the inequalities (16) - (17), (15) yields

$$\begin{aligned}\frac{dV(t)}{dt} \leq& e_x^\top(t)\left[\Pi + \mu(K_2C)^\top K_2C - 2\Omega E K_2 C\right]e_x(t)\\ &- e_f^\top(t-\tau)\left[\Xi - \mu K^\top(t)K(t)\right]e_f(t-\tau)\\ &+ e_x^\top(t)\left[2\Omega E - 2\mu(K_2C)^\top\right]K(t)e_f(t-\tau)\\ &+ e_x^\top(t)\left[2\Omega E - 2\mu(K_2C)^\top\right]\tilde{f}(t)\\ &- \xi e_f^\top(t)\Xi e_f(t) + \mu\tilde{f}^\top(t)\tilde{f}(t)\\ &+ 2\mu\tilde{f}^\top(t)K(t)e_f(t-\tau)\\ &- 2\mu\tilde{f}^\top(t)K_2Ce_x(t).\end{aligned} \tag{18}$$

According to Lemma 2, we get

$$\begin{aligned}2\mu\tilde{f}^\top(t)K(t)e_f(t-\tau) \leq& \frac{1}{\phi_1}\left(\mu\tilde{f}^\top(t)\right)^\top\mu\tilde{f}^\top(t)\\ &+ \phi_1 e_f^\top(t-\tau)K^\top(t)K(t)e_f(t-\tau)\\ \leq& \phi_1 e_f^\top(t-\tau)K^\top(t)K(t)e_f(t-\tau)\\ &+ \frac{\mu^2\vartheta^2}{\phi_1}\end{aligned} \tag{19}$$

and

$$\begin{aligned}-2\mu\tilde{f}^\top(t)K_2Ce_x(t) \leq& \frac{1}{\phi_2}\left(\mu\tilde{f}^\top(t)\right)^\top\mu\tilde{f}^\top(t)\\ &+ \phi_2 e_x^\top(t)(K_2C)^\top K_2Ce_x(t)\\ \leq& \frac{\mu^2\vartheta^2}{\phi_2} + \phi_2 e_x^\top(t)(K_2C)^\top K_2Ce_x(t)\end{aligned} \tag{20}$$

where $\phi_1 > 0$ and $\phi_2 > 0$ are constants. Consider condition C3, *i.e.*, $\Omega E = \mu(K_2C)^\top$, then we have

$$\begin{aligned}\frac{dV(t)}{dt} \leq& e_x^\top(t)\left[\Pi - (\mu-\phi_2)(K_2C)^\top K_2C\right]e_x(t)\\ &- e_f^\top(t-\tau)\left[\Xi - (\mu-\phi_1)K^\top(t)K(t)\right]e_f(t-\tau)\\ &- \xi e_f^\top(t)\Xi e_f(t) + \mu^2\vartheta^2\left(\mu + \frac{1}{\phi_1} + \frac{1}{\phi_2}\right).\end{aligned} \tag{21}$$

Let $Q = (\mu-\phi_2)(K_2C)^\top K_2C$, $H = \Xi-(\mu-\phi_1)K^\top(t)K(t)$; therefore, by considering C2 and C3, we obtain

$$\frac{dV(t)}{dt} \leq e_x^\top(t)\left[\Pi - Q\right]e_x(t) - e_f^\top(t-\tau)He_f(t-\tau) - \xi e_f^\top(t)\Xi e_f(t) + \delta \tag{22}$$

with

$$\delta = \mu^2\vartheta^2(\mu + \frac{1}{\phi_1} + \frac{1}{\phi_2}) \tag{23}$$

being a positive constant. Further, denote

$$\eta(t) = \text{col}[e_x^\top(t), e_f^\top(t), e_f^\top(t-\tau)], \tag{24}$$

then (22) becomes

$$\frac{dV(t)}{dt} \leq -\pi\|\eta(t)\|^2 + \delta \tag{25}$$

where $\pi = \lambda_{\max}(-\Pi + Q, H, \xi\Xi)$. It is clear that when $\|\eta(t)\| > \sqrt{\delta/\pi}$, $\frac{dV(t)}{dt} < 0$. Besides, (25) defines a set:

$$\mathcal{A} := \left\{\ell \in \mathbb{R}^{n+2q} \mid -\pi\|\ell\|^2 + \delta > 0\right\}. \tag{26}$$

Denote the common point-to-set distance for $\mathcal{A}$ and $\eta(t)$:

$$d(\eta, \mathcal{A}) = \inf_{c\in\mathcal{A}} \|\eta - c\|. \tag{27}$$

Therefore, (25) also means that the TLI-LO (6) is globally asymptotically stable w.r.t. the set $\mathcal{A}$ and then [9]

$$\lim_{t\to\infty} d(\eta(t), \mathcal{A}) = 0. \tag{28}$$

This completes the proof. $\square$

Remark 7. Theoretically, the selection of τ affects the quantity of ϑ in Lemma 1 and affects the radius of the ball converged to; *i.e.*, the result was shown by (26). A smaller τ can accommodate faster-varying $f(t)$. Since most of practical systems are composed of digital circuits, in general, τ can be chosen for one/multiple sampling intervals.

Remark 8. Since LO parameters must satisfy three conditions in Proposition 1, we can first focus on C3 and choose the appropriate Ξ, $K(t)$, and then obtain a μ. Subsequently, since no $K(t)$ exists in C1 and C2, we can use the traditional Linear Matrix Inequality tool to solve for them. Due to the length of this process, we omit it here. Related methods can be found in [4].

Remark 9. Resource efficiency is an important topic when designing control system schemes [10]–[12]. Data-driven method can be used when computing resources are sufficient [13], [14]. LO is useful to achieve control purpose in scenarios with limited resources such as space vehicles, vessels, unmanned surface vehicles and master-slave systems, [15]–[19].

IV. Comparative Examples

The parameters are given by $A = [0,1;-1,0]$, $B = [1,0;0,1]$, $C = [1,1;0,1]$, $E = [1,1]^\top$, $L = [1,0;0,2]$, and $K_2 = [10,10]$. The initial observer state is $\hat{x}(0) = [0,0]^\top$. The learning interval in (6) is $\tau = 0.01$s. The initial system state in (1) is $x(0) = [0.1, 0.2]^\top$. The TLI parameters in (9) are $\lambda_1 = 4$, $\lambda_2 = 2$, and $\gamma = 0.001$. To verify the ability of TLI-LO to cope with **constant (case #1)** and **time-varying (case #2)** uncertainty signals and its superiority to existing studies, we conducted two comparative simulations. The comparative one should be when K is *fixed* using the abbreviation FTI-LO in the simulation results presentation. Based on the law of control variables, we chose the FLI-LO scheme with the same parameters except for the fixed $k_i = 0.995$ (typical setting in [7, Section 5.1]). Figs. 1-5 present the comparative results. Figs. 1-2 exhibit the results for a constant uncertainty (a step signal of amplitude 0.1 appears at 2s), and Figs. 3-4 for a time-varying uncertainty (using a sine-wave signal of amplitude 0.1). Simulation results in the upper part of each figure are for TLI-LO and in the lower part for FLI-LO. From the above figures, it is clear that a twofold conclusion can be made:

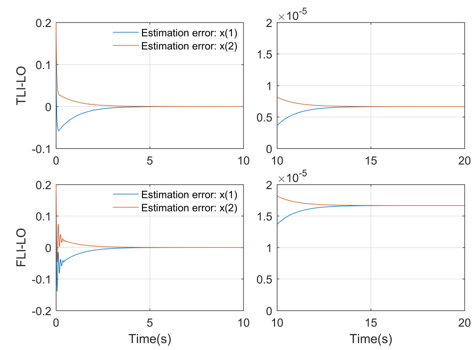

Fig. 1. State estimation errors in case #1 (constant uncertainty).

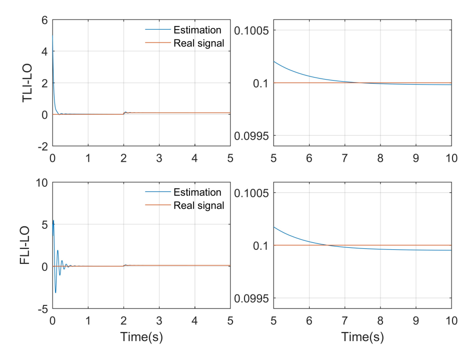

Fig. 2. Uncertainty estimation errors in case #1 (constant uncertainty).

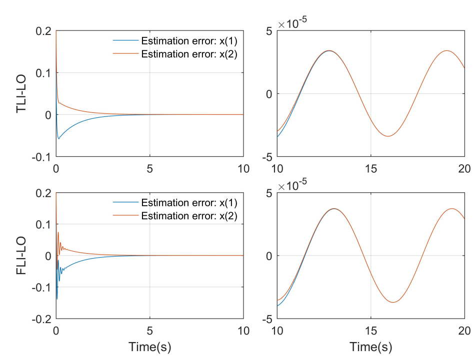

Fig. 3. State estimation errors in case #2 (time-varying uncertainty).

1) The chattering behavior of FLI-LO is significantly attenuated in TLI-LO; thus, the design goal is achieved.
2) The estimation accuracy of TLI-LO maintains a high level for both constant and time-varying uncertain signals.

Fig. 5 provides the variation of TLI; it can be seen that the learning intensity is small when the estimation error and its variation are relatively large, and it converges to 1 when they are small. Meanwhile, there is also a noticeable variation in the learning intensity when the step signal appears at 2s. Consistently, this corresponds to the fact that TLI-LO can attenuate the chattering response.

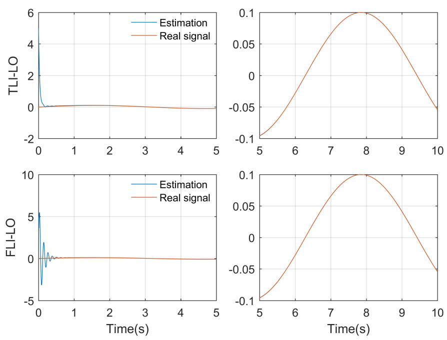

Fig. 4. Uncertainty estimation errors in case #2 (time-varying uncertainty).

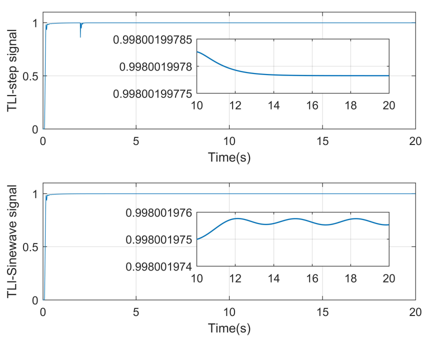

Fig. 5. Evolution of time-varying learning intensity in both simulations.

V. Conclusions

This brief paper presents a new TLI-LO for state and uncertainty identification with a weakened chattering response. The proposed TLI approach tunes the learning intensity with different error conditions, attenuating the common chattering issue in conventional LOs with FLI and thus enhancing the estimation performance. A noticeable performance improvement can be seen from the simulation. The TLI approach shows satisfactory results to improve LO designs, which will promote our future exploration.

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